## The kaon form factor in the light-cone quark model

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**Abstract.** The electromagnetic form factor of the kaon meson is calculated in the light-cone formalism of the relativistic constituent quark model. The calculated  $K^+$  form factor is consistent with almost all of the available experimental data at low-energy scale, while other properties of the kaon could also be interrelated in this representation with reasonable parameters. Predictions of the form factors for the charged and neutral kaons at a higher-energy scale are also given, and we find the non-zero  $K^0$  form factor at  $Q^2 \neq 0$  due to the mass difference between the strange and down quarks inside  $K^0$ .

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The light-cone formalism [1,2] provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom. The electromagnetic form factor of the pion has been studied and discussed [3–5] in the light-cone formalism, which has proved successful in explaining the experimental data. Similar to the pion, the kaon is also composed by two quarks, but with different quark masses. Therefore, it becomes a little more complicated to obtain the light-cone wave function of the kaon and to compute the kaon space-like form factor. However, unlike the  $\pi^0$ , which has zero form factor due to its quark and antiquark with opposite charges (*i.e.*, a pair of quark and antiquark of the same flavor), the  $K^0$ form factor will be non-zero due to the different contributions from the strange  $(\overline{s})$  and down (d) quarks inside  $K^0$ . Thus, measurements of the form factors of the charged kaon  $(K^{\pm})$  and neutral kaon  $(K^0 \text{ and } \overline{K}^0)$  will provide more information concerning the internal structure of the mesons.

In order to obtain the light-cone spin space wave function of the kaon, we transform the ordinary instant-form SU(6) quark model space wave function of the kaon into the light-cone dynamics [3,6–8]. In the kaon rest frame  $(q_1 + q_2 = 0)$ , the instant-form spin space wave function of the kaon is

$$\chi_T = \left(\chi_1^{\uparrow}\chi_2^{\downarrow} - \chi_2^{\uparrow}\chi_1^{\downarrow}\right) \Big/ \sqrt{2},\tag{1}$$

in which  $\chi_i^{\uparrow,\downarrow}$  is the two-component Pauli spinor and the two quarks have 4-momentum  $q_1^{\mu} = (q_1^0, \mathbf{q})$  and  $q_2^{\mu} = (q_2^0, -\mathbf{q})$ , with  $q_i^0 = (m_i^2 + \mathbf{q}^2)^{1/2}$ , respectively. The instantform spin states  $|J, s\rangle_T$  and the light-cone form spin states  $|J, \lambda\rangle_F$  are related by a Wigner rotation  $U^J$  [9]

$$|J,\lambda\rangle_F = \sum_s U^J_{s\lambda} |J,s\rangle_T.$$
 (2)

This rotation is called the Melosh rotation [10] for spin-(1/2) particles. Applying the transformation eq. (2) on both sides of eq. (1), we can obtain the spin space wave function of the kaon in the infinite-momentum frame. For the left side, *i.e.*, the kaon, the transformation is simple since the Wigner rotation is unity. For the right side, *i.e.*, two spin-(1/2) partons, the instant-form and light-front form spin states are related by the Melosh transformation [9–11],

$$\chi_{1}^{\uparrow}(T) = \omega_{1} \left[ (q_{1}^{+} + m_{1})\chi_{1}^{\uparrow}(F) - q_{1}^{R}\chi_{1}^{\downarrow}(F) \right],$$
  

$$\chi_{1}^{\downarrow}(T) = \omega_{1} \left[ (q_{1}^{+} + m_{1})\chi_{1}^{\downarrow}(F) + q_{1}^{L}\chi_{1}^{\uparrow}(F) \right],$$
  

$$\chi_{2}^{\uparrow}(T) = \omega_{2} \left[ (q_{2}^{+} + m_{2})\chi_{1}^{\uparrow}(F) - q_{2}^{R}\chi_{2}^{\downarrow}(F) \right],$$
  

$$\chi_{2}^{\downarrow}(T) = \omega_{2} \left[ (q_{2}^{+} + m_{2})\chi_{1}^{\downarrow}(F) + q_{2}^{L}\chi_{2}^{\uparrow}(F) \right],$$
(3)

where  $\omega_i = [2q_i^+(q_i^0 + m_i)]^{-1/2}$ ,  $q_i^{R,L} = q_i^1 \pm q_i^2$ , and  $q_i^+ = q_i^0 + q_i^3$ . Then we get the light-cone spin wave function for

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the kaon,

$$\chi^{K}(x,\mathbf{k}_{\perp}) = \sum_{\lambda_{1},\lambda_{2}} C_{0}^{F}(x,\mathbf{k}_{\perp},\lambda_{1},\lambda_{2})\chi_{1}^{\lambda_{1}}(F)\chi_{2}^{\lambda_{2}}(F), \quad (4)$$

where the component coefficients  $C_{J=0}^F(x, \mathbf{k}_{\perp}, \lambda_1, \lambda_2)$ , when expressed in terms of the instant-form momentum  $q_{\mu} = (q^0, \mathbf{q})$ , have the forms

$$C_{0}^{F}(x, \mathbf{k}_{\perp}, \uparrow, \downarrow) = \omega_{1}\omega_{2} \left[ (q_{1}^{+} + m_{1})(q_{2}^{+} + m_{2}) - \mathbf{q}_{\perp}^{2} \right] / \sqrt{2},$$

$$C_{0}^{F}(x, \mathbf{k}_{\perp}, \downarrow, \uparrow) = -\omega_{1}\omega_{2} \left[ (q_{1}^{+} + m_{1})(q_{2}^{+} + m_{2}) - \mathbf{q}_{\perp}^{2} \right] / \sqrt{2},$$

$$C_{0}^{F}(x, \mathbf{k}_{\perp}, \uparrow, \uparrow) = \omega_{1}\omega_{2} \left[ (q_{1}^{+} + m_{1})q_{2}^{L} - (q_{2}^{+} + m_{2})q_{1}^{L} \right] / \sqrt{2},$$

$$C_{0}^{F}(x, \mathbf{k}_{\perp}, \downarrow, \downarrow) = \omega_{1}\omega_{2} \left[ (q_{1}^{+} + m_{1})q_{2}^{R} - (q_{2}^{+} + m_{2})q_{1}^{R} \right] / \sqrt{2},$$
(5)

which satisfy the relation

$$\sum_{\lambda_1,\lambda_2} C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)^* C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) = 1.$$
(6)

We can see that there are also two higher-helicity  $(\lambda_1 + \lambda_2 = \pm 1)$  components in the expression of the lightcone spin wave function of the kaon besides the ordinaryhelicity  $(\lambda_1 + \lambda_2 = 0)$  components. Such higher-helicity components [3–5] come from the Melosh rotation, and the same effect plays an important role to understand the proton "spin puzzle" in the nucleon case [11,12].

Furthermore, we still have to know the space wave function. Unfortunately, there is no exact solution of the Bethe-Salpeter equation for the kaon at present. Approximately, we can adopt the commonly used harmonicoscillator wave function

$$\varphi(q^2) = A \exp[-q^2/2\beta^2], \qquad (7)$$

which is a non-relativistic solution of the Bethe-Salpeter equation in an instantaneous approximation in the rest frame for meson [13]. By assuming that the relation between the instant-form momentum  $\mathbf{q} = (q^3, \mathbf{q}_{\perp})$  and the light-cone momentum  $\underline{k} = (x, \mathbf{k}_{\perp})$  is by no means unique, and according to the light-cone formalism, we construct models to relate them. In this presentation, we adopt the connection [6–8] in the light-front dynamics:

$$\begin{aligned} x_1 M &= q_1^0 + q_1^3, \\ x_2 M &= q_2^0 + q_2^3, \\ \mathbf{k}_\perp &= \mathbf{q}_\perp, \end{aligned}$$
 (8)

here  $x_i$  (i = 1, 2), with  $x_1 + x_2 = 1$ , is the light-cone momentum fraction of the quark in the 2-particle Fock state. In the rest frame  $(q_1 + q_2 = 0)$ , from eq. (8) we can find that M satisfies

$$M^{2} = \frac{m_{1}^{2} + \mathbf{k}_{\perp}^{2}}{x_{1}} + \frac{m_{2}^{2} + \mathbf{k}_{\perp}^{2}}{x_{2}}.$$
 (9)

If we let  $x_1 = x$ , then we can get  $x_2 = 1 - x$ . Then eq. (9) can be written as follows:

$$M^{2} = \frac{m_{1}^{2} + \mathbf{k}_{\perp}^{2}}{x} + \frac{m_{2}^{2} + \mathbf{k}_{\perp}^{2}}{1 - x}.$$
 (10)

From eq. (8) we can also obtain

$$q_1^0 = \frac{1}{2}Mx + \frac{\mathbf{k}_{\perp}^2 + m_1^2}{2xM},$$
  

$$q_1^3 = \frac{1}{2}Mx - \frac{\mathbf{k}_{\perp}^2 + m_1^2}{2xM},$$
(11)

$$q_2^0 = \frac{1}{2}M(1-x) + \frac{\mathbf{k}_{\perp}^2 + m_2^2}{2(1-x)M},$$
  

$$q_2^3 = \frac{1}{2}M(1-x) - \frac{\mathbf{k}_{\perp}^2 + m_2^2}{2(1-x)M}.$$
(12)

From eq. (11) and eq. (12) we can easily find that  $q_1^3 = -q_2^3$ . Thus, we have

$$q_1^+ = xM,$$
  
 $q_2^+ = (1-x)M,$  (13)

$$2q_1^+(q_1^0 + m_1) = (xM + m_1)^2 + \mathbf{k}_{\perp}^2, 2q_2^+(q_2^0 + m_2) = [(1 - x)M + m_2]^2 + \mathbf{k}_{\perp}^2.$$
(14)

Then we can get

$$q^{2} = (q_{1})^{2} = (q_{2})^{2} = \frac{1}{4}M^{2} + \frac{(m_{1}^{2} - m_{2}^{2})^{2}}{4M^{2}} - \frac{1}{2}(m_{1}^{2} + m_{2}^{2}),$$
(15)

where

$$M^2 = \frac{m_1^2 + \mathbf{k}_{\perp}^2}{x} + \frac{m_2^2 + \mathbf{k}_{\perp}^2}{1 - x}.$$

There is still another way to obtain eq. (15). Brodsky-Huang-Lepage suggested a connection between the equaltime wave function in the rest frame and the light-cone wave function by equating the off-shell propagator  $\varepsilon = M^2 - (\sum_{i=1}^n k_i)^2$  in the two frames [14]:

$$\varepsilon = \begin{cases} M^2 - (\sum_{i=1}^n q_i^0)^2, \\ \sum_{i=1}^n q_i = 0, \text{ (c.m.)} \\ M^2 - \sum_{i=1}^n \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i}, \\ \sum_{i=1}^n \mathbf{k}_{\perp i} = 0, \qquad \sum_{i=1}^n x_i = 1. \text{ (l.c.)} \end{cases}$$

From the equation above, for two-particle systems one can get

$$q^{2} = \frac{1}{4} \left( \frac{\mathbf{k}_{\perp}^{2} + m_{1}^{2}}{x} + \frac{\mathbf{k}_{\perp}^{2} + m_{2}^{2}}{1 - x} \right) + \frac{(m_{1}^{2} - m_{2}^{2})^{2}}{4 \left( \frac{\mathbf{k}_{\perp}^{2} + m_{1}^{2}}{x} + \frac{\mathbf{k}_{\perp}^{2} + m_{2}^{2}}{1 - x} \right)} - \frac{1}{2} (m_{1}^{2} + m_{2}^{2}).$$
(16)

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Obviously, eq. (15) and eq. (16) are the same, that is to say that, although we have employed different assumptions at the beginning, we obtain the same result of  $q^2$ at last. This may indicate that the model that we have established is self-explained.

By adopting the Brodsky-Huang-Lepage (BHL) prescription [2], we can obtain

$$\varphi_{\text{BHL}} = A_0 \exp\left[-\frac{\frac{\mathbf{k}_{\perp}^2 + m_1^2}{x} + \frac{\mathbf{k}_{\perp}^2 + m_2^2}{1-x}}{8\beta^2} - \frac{(m_1^2 - m_2^2)^2}{8\beta^2 \left(\frac{\mathbf{k}_{\perp}^2 + m_1^2}{x} + \frac{\mathbf{k}_{\perp}^2 + m_2^2}{1-x}\right)}\right],\tag{17}$$

in which we let  $A_0 = A \exp\left[\frac{m_1^2 + m_2^2}{4\beta^2}\right]$ . The contributions from non-zero transversal momentum  $|\mathbf{k}_{\perp}|$  in the endpoint  $x \to 0$  and  $x \to 1$  regions are highly suppressed by the exponential fall-off, so this wave function provides an automatic cut-off on  $|\mathbf{k}_{\perp}|$ . This feature is introduced via the BHL prescription which relies on the free light-cone Hamiltonian.

Therefore, the light-cone wave function for the kaon can be written as follows:

$$\psi = \varphi_{\rm BHL} \chi^K(x, \mathbf{k}_\perp), \tag{18}$$

in which the parameters are the normalization constant  $A_0$ , the harmonic scale  $\beta$  and the quark masses  $m_1$  and  $m_2$ . Thus, we employ the following four constraints to adjust those above four parameters:

1) The normalization condition

$$\int \frac{\mathrm{d}^2 \mathbf{k}_\perp \mathrm{d}x}{16\pi^3} \psi^* \psi = \int \frac{\mathrm{d}^2 \mathbf{k}_\perp \mathrm{d}x}{16\pi^3} \varphi^*_{\mathrm{BHL}} \varphi_{\mathrm{BHL}} = 1, \qquad (19)$$

which is essentially a valence quark dominance assumption [3].

2) The weak-decay constant  $f_K = 113.4$  MeV is defined [15,16] from  $K \to \mu \nu$  decay, thus one obtains

$$\int_{0}^{1} dx \int \frac{d^{2}\mathbf{k}_{\perp}}{16\pi^{3}} \frac{(k_{1}^{+}+m_{1})(k_{2}^{+}+m_{2})-\mathbf{k}_{\perp}^{2}}{\left[(k_{1}^{+}+m_{1})^{2}+\mathbf{k}_{\perp}^{2}\right]^{1/2}\left[(k_{2}^{+}+m_{2})^{2}+\mathbf{k}_{\perp}^{2}\right]^{1/2}} \cdot \varphi_{\rm BHL} = \frac{f_{K}}{2\sqrt{3}}.$$
(20)

3) The charged mean-square radius of  $K^+$  is defined as

$$\langle r_{K^+}^2 \rangle = -6 \frac{\partial F_{K^+}(Q^2)}{\partial Q^2}|_{Q^2=0}.$$
 (21)

We can find the experimental value of  $\langle r^2_{K^\pm}\rangle=0.34\pm0.05$  fm<sup>2</sup> [17], and  $\langle r^2_{K^-}\rangle=0.28\pm0.05$  fm<sup>2</sup> [18].

4) The charged mean-square radius of  $K^0$  is defined as

$$\langle r_{K^0}^2 \rangle = -6 \frac{\partial F_{K^0}(Q^2)}{\partial Q^2}|_{Q^2=0}.$$
 (22)

We can find the experimental value of  $\langle r_{K^0}^2 \rangle = -0.054 \pm \text{where } \mathbf{k}'_{\perp} = \mathbf{k}_{\perp} + (1-x)\mathbf{q}_{\perp}$  is the internal quark transverse  $0.026 \text{ fm}^2$  [19].

Therefore, we can obtain  $m_1 = 500$  MeV (e.g., the strange quark),  $m_2 = 250$  MeV (e.g., the up quark or the down quark, assuming  $m_u = m_d$  ),  $\beta = 393$  MeV and  $A_0 = 0.0742$ . It is interesting to notice that the masses of the strange quarks and the light-flavor quarks from the above constraints are just in the correct range of the constituent quark masses from more general considerations.

Reversely, we can compute the value of  $f_K$ ,  $\langle r_{K^+}^2 \rangle$ , and  $\langle r_{{\scriptscriptstyle K}^0}^2\rangle$  by using the four parameters above

$$f_K = 113.3 \text{ MeV},$$
  
 $\langle r_{K^+}^2 \rangle = 0.30 \text{ fm}^2,$   
 $\langle r_{K^0}^2 \rangle = -0.055 \text{ fm}^2.$  (23)

The results fit the experimental values well. Naturally, the form factor results emerging from this assumption are in quite good agreement with the experimental data. Moreover, the values of the parameters  $(m_1, m_2, \beta)$  are compatible with other quark models [16, 20, 21].

Since the Wigner rotation relating spin state in different frames is unity under kinetic Lorentz transformation in the light-cone formalism, the spin structures of hadrons are the same in different frames related by Lorentz transformation. Therefore, we can calculate the electromagnetic form factor from the Drell-Yan-West formula [22] by using the light-cone formalism

$$F(Q^2) = \sum_{n,\lambda_i} \sum_j e_j \int [\mathrm{d}x] [\mathrm{d}^2 \mathbf{k}_{\perp}] \\ \times \psi_n^*(x_i, \mathbf{k}_{\perp i}, \lambda_i) \psi_n(x_i, \mathbf{k}'_{\perp i}, \lambda_i), \qquad (24)$$

where  $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_{\perp} + \mathbf{q}_{\perp}$  for the struck quark,  $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_{\perp}$  for the spectator quarks,  $[d^2 \mathbf{k}_{\perp}] = d^2 \mathbf{k}_{\perp} / 16\pi^3$ ,  $e_j$  is the electric charge of the struck quark, and the virtual-photon momentum  $q_{\mu}$  is specified with  $q^+ = 0$  to eliminate the Z-graph contributions [1,2,23]. Other choice of  $q_{\mu}$  will cause contributions from Z-graphs, and it should give the same result as that in the case of  $q^+ = 0$  if all the graphs are taken into account [24]. In the light-cone formalism, there is a relation between  $Q^2$  and  $\mathbf{q}_{\perp}^2$ :

$$-Q^2 = q^2 = q^+ q^- - \mathbf{q}_\perp^2.$$
 (25)

Since  $q^+ = 0$ , then according to eq. (25), one can easily get  $\mathbf{q}_{\perp}^2 = Q^2$ .

Because  $K^+ = u\overline{s}$  and  $K^- = s\overline{u}$ , one can find that  $F_{K^+}(Q^2) = -F_{K^-}(Q^2)$  according to eq. (24). Thereby, we just need to calculate the  $K^+$  form factor

$$F_{K^+}(Q^2) = \frac{1}{3}e \int \mathrm{d}x \frac{\mathrm{d}^2 \mathbf{k}_\perp}{16\pi^3} \mathcal{M}_1$$
  
 
$$\times \varphi^*(x, \mathbf{k}_\perp, m_1, m_2)\varphi(x, \mathbf{k}'_\perp, m_1, m_2)$$
  
 
$$+ \frac{2}{3}e \int \mathrm{d}x \frac{\mathrm{d}^2 \mathbf{k}_\perp}{16\pi^3} \mathcal{M}_2$$
  
 
$$\times \varphi^*(x, \mathbf{k}_\perp, m_2, m_1)\varphi(x, \mathbf{k}'_\perp, m_2, m_1), \quad (26)$$

momentum of the struck kaon in the center-of-mass frame,



Fig. 1. The  $K^+$  form factor calculated with the wave function in the BHL prescription at low  $Q^2$  compared with the experimental data. The data are taken from refs. [17] and [18].

and

$$\mathcal{M}_{1} = \frac{(a_{1}a_{2} - \mathbf{k}_{\perp}^{2})(a_{1}'a_{2}' - \mathbf{k}_{\perp}'^{2}) + (a_{1} + a_{2})(a_{1}' + a_{2}')\mathbf{k}_{\perp} \cdot \mathbf{k}_{\perp}'}{\left[(a_{1}^{2} + \mathbf{k}_{\perp}^{2})(a_{2}^{2} + \mathbf{k}_{\perp}'^{2})(a_{1}'^{2} + \mathbf{k}_{\perp}'^{2})(a_{2}'^{2} + \mathbf{k}_{\perp}'^{2})\right]^{1/2}},$$

$$\mathcal{M}_{2} = \frac{(b_{1}b_{2} - \mathbf{k}_{\perp}^{2})(b_{1}'b_{2}' - \mathbf{k}_{\perp}'^{2}) + (b_{1} + b_{2})(b_{1}' + b_{2}')\mathbf{k}_{\perp} \cdot \mathbf{k}_{\perp}'}{\left[(b_{1}^{2} + \mathbf{k}_{\perp}^{2})(b_{2}^{2} + \mathbf{k}_{\perp}'^{2})(b_{1}'^{2} + \mathbf{k}_{\perp}'^{2})(b_{2}'^{2} + \mathbf{k}_{\perp}'^{2})\right]^{1/2}},$$

$$(27)$$

$$\mathcal{M}_{2} = \frac{(b_{1}b_{2} - \mathbf{k}_{\perp}^{2})(b_{1}'b_{2}' - \mathbf{k}_{\perp}'^{2}) + (b_{1} + b_{2})(b_{1}' + b_{2}')\mathbf{k}_{\perp} \cdot \mathbf{k}_{\perp}'}{\left[(b_{1}^{2} + \mathbf{k}_{\perp}^{2})(b_{2}^{2} + \mathbf{k}_{\perp}^{2})(b_{1}'^{2} + \mathbf{k}_{\perp}'^{2})(b_{2}'^{2} + \mathbf{k}_{\perp}'^{2})\right]^{1/2}},$$

$$(28)$$

in which

$$a_{1} = xM_{a} + m_{1}, \quad m_{1} = m_{\overline{s}} = 500 \text{ MeV},$$

$$a_{2} = (1 - x)M_{a} + m_{2}, \quad m_{2} = m_{u} = 250 \text{ MeV},$$

$$a'_{1} = xM'_{a} + m_{1},$$

$$a'_{2} = (1 - x)M'_{a} + m_{2},$$

$$b_{1} = xM_{b} + m_{2},$$

$$b_{2} = (1 - x)M_{b} + m_{1},$$

$$b'_{1} = xM'_{b} + m_{2},$$

$$b'_{2} = (1 - x)M'_{b} + m_{1},$$
(29)

and in which

$$M_{a}^{2} = \frac{m_{1}^{2} + \mathbf{k}_{\perp}^{2}}{x} + \frac{m_{2}^{2} + \mathbf{k}_{\perp}^{2}}{1 - x},$$

$$M_{a}^{\prime 2} = \frac{m_{1}^{2} + \mathbf{k}_{\perp}^{\prime 2}}{x} + \frac{m_{2}^{2} + \mathbf{k}_{\perp}^{\prime 2}}{1 - x},$$

$$M_{b}^{2} = \frac{m_{2}^{2} + \mathbf{k}_{\perp}^{2}}{x} + \frac{m_{1}^{2} + \mathbf{k}_{\perp}^{2}}{1 - x},$$

$$M_{b}^{\prime 2} = \frac{m_{2}^{2} + \mathbf{k}_{\perp}^{\prime 2}}{x} + \frac{m_{1}^{2} + \mathbf{k}_{\perp}^{\prime 2}}{1 - x}.$$
(30)

For the same reason, since  $K^0 = d\overline{s}$  and  $\overline{K}^0 = s\overline{d}$ , one can also find that  $F_{K^0}(Q^2) = -F_{\overline{K}^0}(Q^2)$  according to



**Fig. 2.** Theoretical electromagnetic form factor of  $K^+$  and  $K^0$ , represented by the solid and dashed lines, respectively. The data for  $K^+$  are taken from ref. [18].

eq. (24). Thereby, we just need to calculate the  $K^0$  form factor

$$F_{K^0}(Q^2) = \frac{1}{3}e \int \mathrm{d}x \frac{\mathrm{d}^2 \mathbf{k}_\perp}{16\pi^3} \mathcal{M}_1$$
  
 
$$\times \varphi^*(x, \mathbf{k}_\perp, m_1, m_2)\varphi(x, \mathbf{k}'_\perp, m_1, m_2)$$
  
 
$$-\frac{1}{3}e \int \mathrm{d}x \frac{\mathrm{d}^2 \mathbf{k}_\perp}{16\pi^3} \mathcal{M}_2$$
  
 
$$\times \varphi^*(x, \mathbf{k}_\perp, m_2, m_1)\varphi(x, \mathbf{k}'_\perp, m_2, m_1). \quad (31)$$

The definitions of the  $\mathcal{M}_1$  and  $\mathcal{M}_2$  are the same with the definitions in the calculation of  $K^+$  form factor which we have given above.

Because of the existence of a mass difference between the down quark and the strange quark, we find that  $F_{K^0}(Q^2) \neq 0$  at  $Q^2 \neq 0$ . In comparison, we may note that  $F_{\pi^0}(Q^2) \equiv 0$  because  $\pi^0$  has zero charge and equal quark masses. Naturally, we can figure out that the non-zero  $K^0$ form factor is strongly dependent on the value of the mass difference between the strange quark and the down quark. This aspect is useful to reveal the different contributions from the strange and down quarks inside  $K^0$ .

Figure 1 indicates that in the case of low  $Q^2$ , the theoretical values of the  $K^+$  form factor fit the experimental data very well. The same model can also provide very good description of the charged-pion form factor for  $Q^2 \leq 2$  $(\text{GeV}/c)^2$  [3]. Because we are in lack of experimental data for  $K^+$  in the higher-energy scale, we give the predictions of the  $Q^2 F_K(Q^2)$  values for  $K^+$  and  $K^0$  in fig. 2. Since  $K^0$ has zero charge, we can see that its electromagnetic form factor is much less than that of  $K^+$ . Thus, high precision is needed to measure the  $K^0$  form factor experimentally, since the electroproduction cross-section is small.

It is necessary to point out that this work should be considered as a light-cone version of the relativistic constituent quark model [3,25], and it should be only valid in the low-energy scale of about  $Q^2 \leq 2$  (GeV/c)<sup>2</sup>. Similar works on the kaon have also been given in [20,26]. It is different from the light-cone perturbative QCD approach [2], which is applicable at the high-energy scale of  $Q^2 > 2 \ (\text{GeV}/c)^2$ . The reason is that the hard-gluon exchanges between the quark-antiquark of the meson should be considered at high  $Q^2$ , and this feature is incorporated in the light-cone perturbative QCD approach. An ordinary input wave function may contain uncertainties which invalid the prediction at high  $Q^2$  in the constituent quark model framework. If the constituent quark model prediction is happened to work at a higher  $Q^2$ , it might be by chance or may imply a reasonable input wave function that contains some features simulating the hard-gluon exchanges. So the agreement of a constituent quark model prediction with experiments might serve as a support of the applicability of the input wave function from a low-energy scale to a somewhat higher scale.

In summary, we calculated the electromagnetic form factor of the kaon by adopting the light-cone formalism of the relativistic constituent quark model. By adjusting the parameters through the experimental values of the weakdecay constant and the charged mean-square radius, the model can give a good fit to the available experimental values of kaon form factors. We also predicted the form factors for both charged and neutral kaons,  $K^{\pm}$  and  $K^0$ . We expect the predictions to be valid when  $Q^2 \leq 2$  (GeV/c)<sup>2</sup> at a low-energy scale. A non-zero form factor of  $K^0$  is predicted at  $Q^2 \neq 0$ , and it will be useful to reveal the different contributions from strange and down quarks inside  $K^0$ .

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## References

- S.J. Brodsky, in Lectures on Lepton Nucleon Scattering and Quantum Chromodynamics, edited by A. Jaffe, D. Ruelle (Birkhäuser, Boston, 1982) p. 255; S.J. Brodsky, G.P. Lepage, in Perturbative Quantum Chromodynamics, edited by A.H. Mueller (World Scientific, Singapore, 1989) p. 93; S.J. Brodsky, H-C. Pauli, S.S. Pinsky, Phys. Rep. **301**, 299 (1998).
- 2. G.P. Lepage, S.J. Brodsky, Phys. Rev. D 22, 2157 (1980).
- 3. B.-Q. Ma, Z. Phys. A **345**, 321 (1993).
- B.-Q. Ma, T. Huang, J. Phys. G: Nucl. Part. Phys. 21, 765 (1995).
- F.-G. Cao, J. Cao, T. Huang, B.-Q. Ma, Phys. Rev. D 55, 7107 (1997).

- M.V. Terent'ev, Yad. Fiz. 24, 207 (1976) (Sov. J. Nucl. Phys. 24, 106 (1976)).
- 7. V.A. Karmanov, Nucl. Phys. B 166, 378 (1980).
- P.L. Chung, F. Coester, W.N. Polyzou, Phys. Lett. B 205, 545 (1988).
- 9. E. Wigner, Ann. Math. 40, 149 (1939).
- 10. H.J. Melosh, Phys. Rev. D 9, 1095 (1974).
- B.-Q. Ma, J. Phys. G: Nucl. Part. Phys. **17**, L53 (1991);
   B.-Q. Ma, Q.-R. Zhang, Z. Phys. C **58**, 479 (1993).
- B.-Q. Ma, Phys. Lett. B **375**, 320 (1996); B.-Q. Ma,
   A. Schäfer, Phys. Lett. B **378**, 307 (1996); B.-Q. Ma,
   I. Schmidt, J. Soffer, Phys. Lett. B **441**, 461 (1998); B.-Q. Ma, I. Schmidt, J.-J. Yang, Eur. Phys. J. A **12**, 353 (2001).
- See, e.g., Elementary Particle Theory Group, Peking University, Acta Phys. Sin. 25, 415 (1976); N. Isgur, in *The New Aspects of Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1980) p. 107.
- S.J. Brodsky, T. Huang, G.P. Lepage, in *Particles and Fields-2, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981*, edited by A.Z. Capri, A.N. Kamal (Plenum, New York, 1983) p. 143. See, also, Tao Huang, Bo-Qiang Ma, Qi-Xing Shen, Phys. Rev. D 49, 1490 (1994).
- T. Azemoon *et al.*, Nucl. Phys. B **95**, 77 (1975);
   S.R. Amendolia *et al.*, Nucl. Phys. B **277**, 168 (1986).
- W.W. Buck, R.A. Williams, Hiroshi Ito, Phys. Lett. B 351, 24 (1995).
- 17. S.R. Amendolia et al., Phys. Lett. B 178, 435 (1986).
- 18. E.B. Dally *et al.*, Phys. Rev. Lett. **45**, 232 (1980).
- 19. W.R. Molzon et al., Phys. Rev. Lett. 41, 1213 (1978).
- H.-M. Choi, C.-R. Ji, Phys. Rev. D 59, 034001; 074015 (1999).
- 21. X.-H. Guo, T. Huang, Phys. Rev. D 43, 2391 (1991).
- S.D. Drell, T.-M. Yan, Phys. Rev. Lett. 24, 181 (1970); G. West, Phys. Rev. Lett. 24, 1206 (1970).
- B.-Q. Ma, PhD Thesis, Peking University (1989);
   B.-Q. Ma, J. Sun, J. Phys. G: Nucl. Part. Phys. 16, 823 (1990);
   B.-Q. Ma, Phys. Rev. C 43, 2821 (1991).
- 24. M. Sawicki, Phys. Rev. D 46, 474 (1992).
- See, e.g., L.A. Kondratyuk, M.V. Terent'ev, Yad. Fiz 31, 1087 (1980) (Sov. J. Nucl. Phys. 31, 561 (1980));
   P.L. Chung, F. Coester, B.D. Keister, W.N. Polyzou, Phys. Rev. C 37, 2000 (1988); H.J. Weber, Ann. Phys. (N.Y.) 207, 417 (1991); W. Jaus, Phys. Rev. D 41, 3394 (1990); 44, 2851 (1991); P.L. Chung, F. Coester, Phys. Rev. D 44, 229 (1991); F. Schlumpf, Phys. Rev. D 48, 4478 (1993); F. Schlumpf, S.J. Brodsky, Phys. Lett. B 360, 1 (1995); B.-Q. Ma, D. Qing, I. Schmidt, Phys. Rev. C 65, 035205 (2002).
- F. Cardarelli *et al.*, Phys. Lett. B **332**, 1 (1994); **357**, 267 (1995); Phys. Rev. D **53**, 6682 (1995); Phys. Rev. C **62**, 065201 (2000) and references therein.